

# A Semi-Unsteady-State Wellbore Steam/Water Flow Model for Prediction of Sandface Conditions in Steam Injection Wells

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## Abstract

A numerical nonisothermal two-phase wellbore model is developed to simulate downward flow of a steam and water mixture in the wellbore. This model entails simultaneous solution of coupled mass and momentum conservation equations inside the wellbore with an energy conservation equation for the fluids within the wellbore, surrounding medium and formation. A new drift-flux model that accounts for slip between the phases inside the wellbore is employed. In addition, a 2D implicit scheme that allows for heat transfer in both the axial and radial directions in the formation is developed. Furthermore, a rigorous nonlinear temperature- and depth-dependent overall heat transfer coefficient is implemented. The model predictions are validated against real field data and other available models. The model is useful for designing well completion and accurately computing the wellbore/formation heat transfer, which is important for estimating oil recovery by using steam injection.

## Introduction

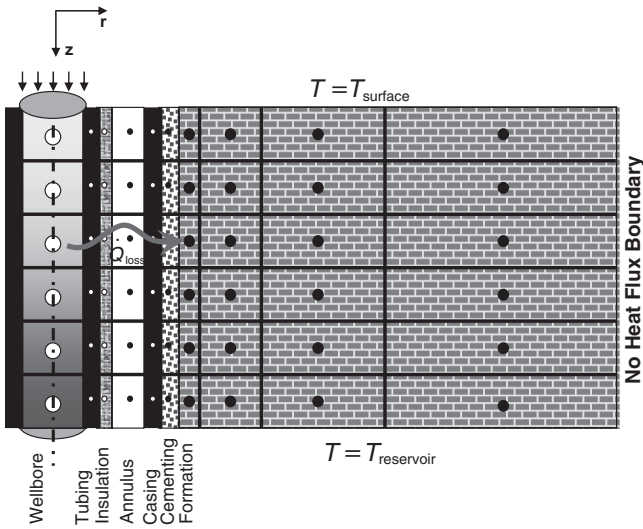
Modelling of steam injection wells for continuous estimation of pressure, temperature and different phase velocities and densities as functions of depth and time is crucial for well design, steam injection projects, planning and data gathering for continuous reservoir management and real-time well monitoring. Once steam is injected in the well, both pressure and temperature of the injected steam and accordingly the densities of water and steam phases will change. These changes are caused by the heat exchange between the steam and cold formation surrounding the well, the friction between the steam and inner tubing surface and the change of the hydrostatic pressure with respect to depth. More importantly, the injected steam quality will drop because of the heat loss from the wellbore system toward the cold formation. The steam quality at the formation can be much worse than that at the wellhead because of an improper wellbore design, no tubular insulation and/or the deep well location. The multiphase nature of the flow inside the wellbore, the complex heat transfer mechanisms between the wellbore and the surrounding medium and the unsteady-state nature of the flow and transport processes make the entire system intricately coupled and extremely difficult to solve.

Numerous investigators have worked on the modelling of both injection and production wells. Among the first papers goes back to Ramey<sup>(1)</sup> in 1962, which has been referred to by many subsequent works modelling the wellbore heat loss and pressure drop. In that paper, the author simplified the heat balance equation to solve it analytically. The steady-state flow of incompressible single phase, with fixed fluid and formation properties with respect to depth and temperature, was analyzed. A simple procedure was pre-

sented to couple the steady-state heat loss of the wellbore fluid with a transient heat flow in the formation by an overall heat transfer coefficient. Moreover, it was assumed that the overall heat transfer coefficient was independent of depth, and the frictional loss and kinetic energy effect were neglected. Ramey<sup>(1)</sup> could derive his two final analytical expressions, one for oil and the other for gas temperature distribution, along the tubing by introducing a term which he referred to as a "time function." This function, however, was assumed to be independent of depth and restricted to some approximations. In 1965, Satter<sup>(2)</sup> improved Ramey's analytical model by considering a depth-dependent overall heat transfer coefficient and phase- and temperature-dependent fluid properties. A year later, Holst and Flock<sup>(3)</sup> added the friction loss and kinetic energy effect to Ramey's and Satter's models. In 1967, Wilhite<sup>(4)</sup> proposed a method for the estimation of an overall heat transfer coefficient that has been widely used and will also be used in this paper.

Pacheco and Ali<sup>(5)</sup> and Herrera et al.<sup>(6)</sup> proposed wellbore models for simulation of a steam injection process and validated their models with field data. They also implemented Wilhite's<sup>(4)</sup> method for the calculation of the overall heat transfer coefficient and Ramey's<sup>(1)</sup> model for the estimation of 1D radial heat loss. In 1980, Shiu and Beggs<sup>(7)</sup> proposed an empirical correlation for oil producing wells to estimate the time function which Ramey defined. This correlation approximates the already approximated Ramey's time function.

In 1981, 1982 and 1985, respectively, Ali<sup>(8)</sup>, Fontanilla and Aziz<sup>(9)</sup> and Yao<sup>(10)</sup> presented two simultaneous ordinary differential equations for estimating the steam pressure and quality, and solved these equations by using the fourth-order Runge-Kutta method. The major differences between these models were the type of correlations used to describe the multiphase flow inside the wellbore and the techniques to evaluate the formation temperature. In 1989, Sharma et al.<sup>(11)</sup> modified Ramey's model for production wells with a downhole electrical heater, and in 1990, Wu and Pruess<sup>(12)</sup> suggested an analytical solution for wellbore heat transmission in a layered formation with different thermal properties without Ramey's assumptions. In 1991, Sagar et al.<sup>(13)</sup> presented a simplified two-phase method for hand calculations using field data. In a 1992 comparative study, Alves et al.<sup>(14)</sup> reported that all existing models suffered from serious assumptions on the thermodynamic behaviour of fluids, and thus were applicable only for limited operational strategies. These authors developed a unified equation for temperature prediction inside the wellbore. In 1994, Hasan and Kabir<sup>(15)</sup> developed an analytical model to determine the flowing fluid temperature inside the well. They started with a steady-state energy balance equation and combined it with the definition of fluid enthalpy in terms of heat capacity and the Joule-Thompson coefficient. Using some simplifications, they then converted the original partial differential equation to an ordinary differential equation and solved it with appropriate boundary conditions. They have been modifying their original model in several recent publications in 2002<sup>(16)</sup>, 2005<sup>(17)</sup>



**Fig. 1: Schematic representation of the discretized wellbore system and formation, and formation boundary conditions with geometric spacing in radial direction for the formation part.**

and 2007<sup>(18)</sup>. In 2004, Hagoort<sup>(19)</sup> did a comprehensive study on Ramey's model in order to find applicable scenarios for this model. Many researchers (including Hagoort) found that Ramey's model works for late times' (more than a week) temperature estimation, but can cause serious errors for early time temperature distribution. In 2008 and 2010, Livescu et al.<sup>(20–22)</sup> developed a comprehensive numerical nonisothermal multiphase wellbore model. After their initial attempts to solve the fully coupled conservation equations, they decoupled the wellbore energy balance equation from the mass balance equation in most of their investigations. They reported that the decoupling can be justified when the change in density of each phase with respect to temperature is much less than that with respect to pressure. Additionally, they found that this decoupling approach can decrease the computational time of the simulations without violating stability. They further showed that if several simplifying assumptions were imposed, their model reduced to Ramey's model.

This paper presents a numerical transient wellbore model for computing the wellbore fluid temperature, pressure, density and velocity profiles in steam injection wells. This model couples mass, momentum and energy balance equations and provides all the necessary data in the well with respect to depth and time for a predetermined surface condition. While the model is designed for steam injection wells, with some minor modifications it can be extended to modelling the injection of other fluids (e.g., hot water injection). This model has some important features for accurate and fast prediction of wellbore conditions. First, a new drift-flux model [Hasan et al.<sup>(23)</sup>] is incorporated into the model for multiphase flow that enables this model to capture the slip phenomenon between the phases. Second, instead of using a fully implicit treatment for the entire wellbore system, we solve this system by using a sequential solution procedure that, in addition to its stability and programming simplicity, increases the speed of simulation. To stabilize the numerical solution procedure, the formation part is solved with a fully implicit scheme that enables us to use irregular grids. Third, a simple iterative procedure is incorporated into the model to predict accurately the depth and time dependence of the overall heat transfer coefficient. Validation and predictive capabilities of this model are determined through comparisons with both field data and other available models.

## Model Formulation

In this section, the problem and all the corresponding formulations are defined. Then in the next section, the solution scheme for solving this complex problem is presented. The entire wellbore/formation system to be solved can be divided into three parts (**Fig. 1**).

### First Part: Wellbore Tubing

Wellbore tubing involves the downward flow of a steam and water mixture. In this part of the wellbore system, the governing equations include a mass balance equation for the water component, an energy balance equation and an equation for the pressure drop. The unknown parameters are the saturation pressure,  $p_{sat}$ , saturation temperature,  $T_{sat}$  ( $p_{sat}$  and  $T_{sat}$  are related because two-phase flow for one component exists), superficial velocity of each phase,  $v_{sL}$  and  $v_{sg}$ , in situ gas volume fraction,  $f_g$ , density of each phase,  $\rho_L$  and  $\rho_g$ , enthalpy of each phase,  $h_L$  and  $h_g$ , and heat loss rate per unit depth to the surrounding medium,  $\dot{Q}_{loss}$ .

### Mass Balance Equation

Because only one component (water) and two phases (liquid water and steam) are present in the tubing, the mass conservation equation is given by

$$-\frac{\partial}{\partial z}(\rho_g v_{sg} + \rho_L v_{sL}) = \frac{\partial}{\partial t}(\rho_m), \quad \dots \quad (1)$$

where  $\rho_m$  is the mixture density, defined as  $\rho_m = f_g \rho_g + (1 - f_g) \rho_L$ , and the in-situ gas volume fraction  $f_g$  is the ratio between the area occupied by the gas phase (steam) and the total cross-sectional area of the tubing ( $f_g = A_g/A_{ti}$ ,  $f_g + f_L = 1$ ). The superficial velocity of each phase is defined by the product of the in-situ phase volume fraction and phase velocity ( $v_{sg} = f_g v_g$ ,  $v_{sL} = f_L v_L$ ). The left side in Equation (1) is the convective flux of the water and steam mixture and the right side is the mass accumulation of the water component (liquid water and steam).

Steam quality is the most important parameter in the steam injection wells and can be easily related to the expressions that were previously defined by:

$$x = \frac{v_{sg} \rho_g}{v_{sg} \rho_g + v_{sL} \rho_L} = \frac{f_g v_g \rho_g}{f_g v_g \rho_g + (1 - f_g) v_L \rho_L} \quad \dots \quad (2)$$

### Momentum Balance Equation

The total pressure drop inside the tubing is the sum of the pressure drops caused by hydrostatic, frictional and acceleration (kinetic) effects<sup>(8,16–18,20–24)</sup>:

$$-\frac{dp}{dz} = -\frac{\rho_m g \cos(\theta)}{g_c} + \frac{f_m v_m^2 \rho_m}{2 d_{ti} g_c} + \frac{\rho_m v_m}{g_c} \frac{dv_m}{dz}, \quad \dots \quad (3)$$

where  $g$  is the gravitational acceleration,  $\theta$  is the local angle between the well and the vertical direction,  $g_c$  is the gravitational conversion constant,  $d_{ti}$  is the inner diameter of the tubing,  $v_m$  is the mixture velocity ( $v_m = v_{sL} + v_{sg}$ ), and  $f_m$  is a friction factor (an empirical factor that depends on the pipe roughness and Reynolds number).

### Energy Balance Equation

The energy balance equation for multiphase flow at the steady-state condition can be expressed as follows<sup>(8,16–18,20–22)</sup>:

$$\frac{\dot{Q}_{loss}}{3600 \times A_{ti}} + \frac{\partial}{\partial t} \left\{ \sum_p \left[ f_p \rho_p \left( u_p + \frac{v_p^2}{2 g_c J_c} \right) \right] \right\} = -\frac{\partial}{\partial z} \left\{ \sum_p \left[ \rho_p v_{sp} \left( h_p + \frac{1}{2} \frac{v_p^2}{g_c J_c} \right) \right] \right\} + \sum_p \frac{\rho_p v_{sp} g \cos(\theta)}{g_c J_c}, \quad \dots \quad (4)$$

where  $A_{ti}$  is the inner tubing area,  $p$  is the phase index (liquid or gas),  $J_c$  is the mechanical equivalent of heat (788 ft-lbf/Btu), and 3,600 converts hour to seconds. In this equation the conductive heat transport (it may become important in the shut-in wells) and the work done on the fluids by the viscous force are assumed to be small and thus ignored. The first term on the right side is an energy flux caused by convection and the work done by the pressure force,

and the second term is the work done on the fluids by the gravitational force. The first term on the left side is the rate of heat loss to the surroundings and the second term is the energy accumulation.

## Second Part: Series of Heat Flow Resistances From Wellbore Tubing to Formation

These resistances include the tubing wall, possible insulation around the tubing, annular space (possibly filled with a gas or liquid, but is sometimes a vacuum), casing wall and cementing behind the casing, as illustrated in Fig. 1. The heat lost from the fluids will go through this series of heat resistances and be finally absorbed by the cold formation that surrounds the wellbore system. In other words,  $\dot{Q}_{loss}$  is the linkage between the wellbore system and the formation. The heat loss can be expressed as

$$\dot{Q}_{loss} = 2\pi r_{to} U_{to} (T_f - T_{wb}), \quad (5)$$

where  $r_{to}$  is the outer tubing radius,  $U_{to}$  is the overall heat transfer coefficient based on  $r_{to}$ ,  $T_f$  is the fluid temperature inside the tubing and  $T_{wb}$  is the temperature at the boundary of the wellbore and formation (i.e., the temperature behind the cementing and start of the formation). The overall heat transfer coefficient is given by

$$\frac{1}{U_{to}} = \frac{r_{to}}{r_{ti} h_f} + \frac{r_{to} \ln(r_{to}/r_{ti})}{k_i} + \frac{r_{to} \ln(r_{ins}/r_{to})}{k_{ins}} + \frac{r_{to}}{r_{ins} (h_c + h_r)} + \frac{r_{to} \ln(r_{co}/r_{ci})}{k_{cas}} + \frac{r_{to} \ln(r_{wb}/r_{co})}{k_{cem}}, \quad (6)$$

where the various parameters are defined in the nomenclature. The convective heat transfer coefficient for the steam/water mixture,  $h_f$ , is usually high [500–2,000 Btu/(hr ft<sup>2</sup> °F)<sup>(4)</sup>]; therefore, the first term on the right side can be ignored and it is assumed that the tubing fluid temperature is equal to the tubing inner-wall temperature. The other remaining terms are heat conduction transfers, except the annulus part that involves both convection and radiation (depending on both the type of gas or liquid that exists in the annulus and the annulus pressure—each plays an important role in the heat loss; in the case of a vacuumed annulus, only radiation will be present). Conduction can also be present in the annular space; its effect, however, has been absorbed into the convective heat transfer coefficient,  $h_c$ . The radiation heat transfer coefficient,  $h_r$ , is calculated by the Stefan-Boltzmann law and the convective heat transfer coefficient is attributed to the Grashof and Prandtl numbers.

## Third Part: Formation

The formation surrounds the wellbore system and absorbs the heat from the tubing fluids. A 2D heat conduction transfer formulation in the formation is employed:

$$\frac{1}{r} \frac{\partial}{\partial r} \left( k_{er} r \frac{\partial T_e}{\partial r} \right) + \frac{\partial}{\partial z} \left( k_{ez} \frac{\partial T_e}{\partial z} \right) = \rho_e C_{pe} \frac{\partial T_e}{\partial t}, \quad (7)$$

where  $T_e$  is the formation temperature,  $k_{er}$  is the conduction coefficient in the radial direction,  $k_{ez}$  is the conduction coefficient in the  $z$ -direction,  $\rho_e$  is the formation density and  $C_{pe}$  is the heat capacity of the formation. In the case where  $k_{er} = k_{ez} = k_e$ , this equation can be written as

$$\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T_e}{\partial r} \right) + \frac{\partial^2 T_e}{\partial z^2} = \frac{\partial^2 T_e}{\partial r^2} + \frac{1}{r} \frac{\partial T_e}{\partial r} + \frac{\partial^2 T_e}{\partial z^2} = \frac{1}{\alpha_e} \frac{\partial T_e}{\partial t}, \quad (8)$$

where  $\alpha_e$  is the formation thermal diffusion coefficient:

$$\alpha_e = \frac{k_e}{\rho_e C_{pe}} \quad (9)$$

To solve Equation (7) or (8), one initial and four boundary conditions are needed. The initial condition given by Equation (10) is simply a linear increase in the temperature of the formation from the surface to the reservoir with a slope of  $g_T$  that is the geothermal gradient of the formation:

$$T_{ei} = T_{eiwb} + g_T z \cos(\theta) \quad (10)$$

Knowing the wellhead temperature, geothermal gradient and wellbore depth and inclination, Equation (10) can be used to calculate the reservoir temperature.

The four boundary conditions as depicted in Fig. 1 are given by:

$$T_e = T_{surface} \quad \text{at} \quad z = 0, \quad r \geq r_{wb}, \quad (11)$$

$$T_e = T_{reservoir} \quad \text{at} \quad z = L, \quad r \geq r_{wb}, \quad (12)$$

$$\frac{\partial T_e}{\partial r} = 0 \quad \text{at} \quad 0 \leq z \leq L, \quad r \rightarrow \infty, \quad (13)$$

$$\dot{Q}_{loss} = -2\pi r k_e \frac{\partial T_e}{\partial r} \quad \text{at} \quad 0 \leq z \leq L, \quad r = r_{wb} \quad (14)$$

That is, the top and bottom boundaries of the formation remain at a constant temperature, there is no heat flux at the outer boundary of the formation and the heat flux is prescribed at the cementing/formation interface as the inner boundary condition.

To have a closed system, two more auxiliary equations are necessary. The first one is the equation of state (EOS) that calculates enthalpy, internal energy, and density of each phase:

## EOS:

$$\text{Input } (T, P, \text{Composition}) \rightarrow \text{Output } (\rho_L, \rho_g, h_L, h_g, \text{ and so on}) \quad (15)$$

The second one is the drift-flux model for estimation of the in situ-volume fraction:

$$f_g = \frac{v_{sg}}{C_o v_m - v_{\infty}}, \quad (16)$$

where  $C_o$  is a profile parameter (or distribution coefficient), which describes the effect of the velocity and concentration profiles and depends on the flow regimes and velocity direction (upward or downward and/or cocurrent or countercurrent flow),  $v_m$  is the average velocity of the mixture and  $v_{\infty}$  is the drift velocity of the gas describing the buoyancy effect. We use the drift-flux model of Hasan et al.<sup>(23)</sup> because of its simplicity, continuity and differentiability. Beggs and Brill's method<sup>(24)</sup> is also implemented into the model for comparison of the results.

## Numerical Implementation

A grid system for both the tubing and formation is first explained, and then all the formulas presented in the previous section are discretized using the finite difference method. Finally, the detailed solution algorithm is explained based on a flow diagram (Fig. 2).

As seen from Fig. 3, a staggered grid is used for the tubing part. This means that all of the variables are assigned to the centre of each gridblock, except the superficial and mixture velocities that are assigned to the boundary of the gridblock. Important advantages of staggered grids are that the transport rate across the faces of control volumes can be computed without interpolation of velocity components and that mass is conserved across the boundary of each gridblock. Because the accumulation term in Equation (7) or (8) has a dominant effect on the unsteadiness of the whole system, the accumulation terms in the mass and energy balance equations are neglected for the wellbore part. As a result, the equations will be easier to solve and the resulting simulator will be faster and more stable. Note that although the accumulation terms were neglected, the whole system is still in an unsteady-state condition. As it can be seen from the formulations, the energy balance [Equation (4)] inside the

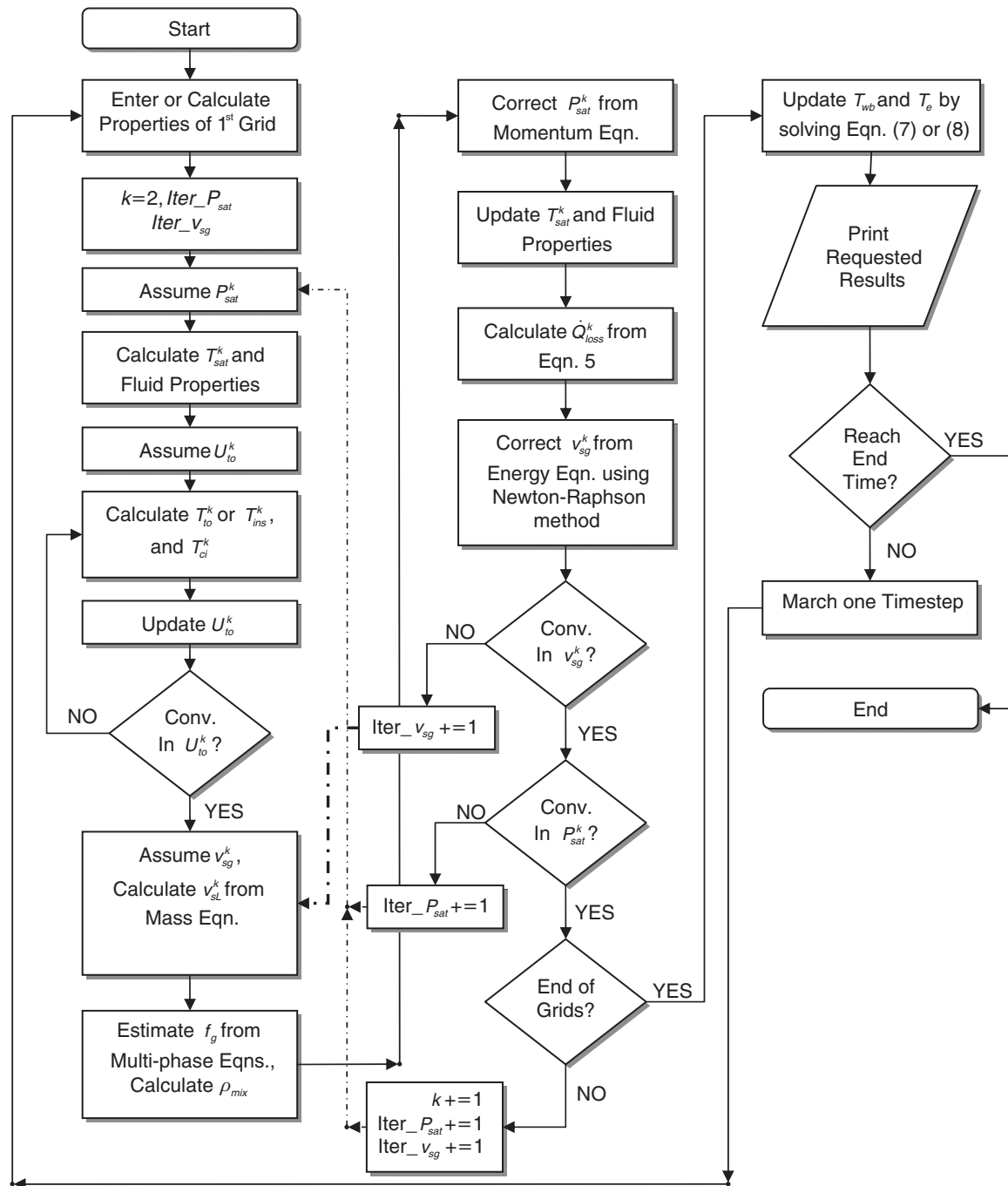


Fig. 2: Main computational and solution-algorithm flowchart.

wellbore is coupled with the earth heat transfer [Equation (8)] by the heat loss [Equation (5)]. Therefore, if the earth temperature changes as the time elapses (which is always the case because the earth is an infinite medium), it will affect the energy balance equation and thus wellbore temperature and consequently, all fluid properties inside the wellbore (because they are functions of temperature). This is why even though the accumulation terms are neglected in the mass and energy balance equations, the fluid properties still change with respect to time. We used the term “semi-unsteady-state” in the title of this paper, which means no accumulation terms for the wellbore formulation, but considers the accumulation term inside the earth. In other words, the whole system was not solved at a fully unsteady state, but at a semi-unsteady-state condition.

For the formation, Equation (8) is discretized with a fully implicit scheme over an irregular grid, where the grid uses an equal spacing in the vertical direction (except near the wellhead and bottomhole

where a more refined grid is used to capture the boundary effects) and a geometric spacing in the radial direction. The accumulation term is considered for the formation and the resulting equation is solved by using BiCGSTAB with an appropriate pre-conditioner.

Now, Equation (1) with the stated assumption can be simplified as

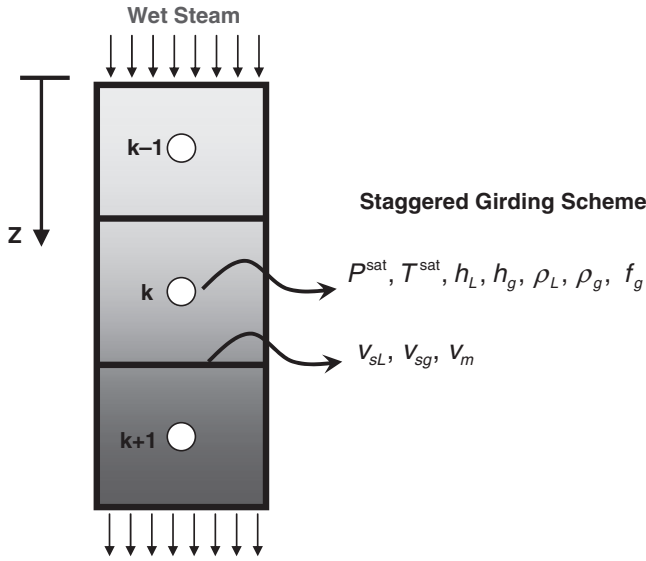
$$\frac{\partial}{\partial z}(\rho_g v_{sg} + \rho_L v_{sL}) = 0, \dots\dots\dots(17)$$

or

$$\rho_g v_{sg} + \rho_L v_{sL} = \frac{\dot{w}}{3600 \times A_{fi}} = \text{constant}$$

$$\Rightarrow v_{sL}|_{k+\frac{1}{2}} = \frac{\dot{w}/(3600 \times A_{fi}) - (\rho_g v_{sg})_{k+\frac{1}{2}}}{\rho_L}|_{k+\frac{1}{2}}, \dots\dots\dots(18)$$





**Fig. 3: Schematic representation of the discretized tubing and staggered grid definition (see the positions where different parameters were defined).**

where  $\dot{w}$  is the total mass injection rate of the steam/water mixture in lbm/hr and the number 3,600 converts hour to seconds. The values of the phase densities at  $k+1/2$  are unknown; they can be found either by an appropriate iteration procedure or the simple upwind approximation:

$$\rho_{k+1/2} \approx \rho_k \dots\dots\dots (19)$$

For the discretization of both the momentum and energy balance equations, Equations (3) and (4), the standard Godunov first-order upwinding scheme is implemented:

$$p_k^n = p_{k-1}^n + \Delta z \left. \frac{dp}{dz} \right|_{k-1/2}^n, \dots\dots\dots (20)$$

$$\begin{aligned} - \left( \frac{dp}{dz} \right)_{k-1/2}^n = & - \frac{\rho_{m_{k-1/2}}^n g \cos(\theta)}{g_c} + \frac{f_{m_{k-1/2}}^n \rho_{m_{k-1/2}}^n (v_{k-1/2}^n)^2}{2d_{ii} g_c} \\ & + \frac{\rho_{m_{k-1/2}}^n v_{m_{k-1/2}}^n}{g_c} \frac{v_{m_k} - v_{m_{k-1}}^n}{z_k - z_{k-1}}, \dots\dots (21) \end{aligned}$$

and

$$\begin{aligned} \frac{\dot{Q}_{loss_k}^n}{3600 \times A_{ti}} = & - \frac{1}{\Delta z \left. \frac{dp}{dz} \right|_{k-1/2}^n} \left\{ \left[ \sum_p \rho_p v_{sp} \left( h_p + \frac{1}{2} \frac{v_p^2}{g_c J_c} \right) \right]_k^n - \right. \\ & \left. \left[ \sum_p \rho_p v_{sp} \left( h_p + \frac{1}{2} \frac{v_p^2}{g_c J_c} \right) \right]_{k-1}^n \right\} + \left[ \sum_p \frac{\rho_p v_{sp} g \cos(\theta)}{g_c J_c} \right]_k^n \dots\dots (22) \end{aligned}$$

The solution of the resulting equations is complicated because of the nonlinear thermodynamical behaviour of steam with the change of the temperature and pressure of the wellbore system with respect to both depth and time. Another complexity is caused by the wellbore and formation heat interaction where the wellbore loses heat to the formation.

As noted, within each timestep for the solution of the formation temperature equation, no accumulation term is considered for the tubing and surrounding system (up to the formation). An iterative procedure that involves updating the nonlinear coefficients and overall heat transfer coefficient at each iteration is used for the entire fluid-flow and heat transport computation. The numerical algo-

rithm works with a double-iterative scheme on the gas' superficial velocity and fluid pressure in order to solve the three conservation equations in a sequential manner. Other internal iterations have been also implemented. These iterations, in addition to the detailed solution procedure, are explained next.

After the model initialization, the main computations are started. This procedure in form of a flowchart is shown in Fig. 2 and is explained in 10 steps as follows:

1. For  $k=1$  (first gridblock)  $\dot{w}$ ,  $P^{sat}$  or  $T^{sat}$  are known (from the well operating conditions) and  $x$ ;  $\rho_L$ ,  $\rho_g$ ,  $v_{sL}$ ,  $v_{sg}$ ,  $h_L$  and  $h_g$  can be calculated with Equation (2) and EOS, respectively.
2. For all other gridblocks at the first iteration, the properties of the previous gridblock are assigned to the current gridblock. If it is not the first iteration, the values of the previous iteration are assigned to the current grid lock (e.g., value of  $P^{sat}$ ).
3. Knowing  $P_k^{sat}$ ,  $T_k^{sat}$ ,  $\rho_L$ ,  $\rho_g$ ,  $h_L$  and  $h_g$  are calculated from EOS.
4. For the estimation of the overall heat transfer coefficient that is a function of the unknown inside-casing temperature and outside tubing (or insulation) temperature, an appropriate iterative procedure is employed to estimate this coefficient as a function of both time and depth. For the first iteration,  $U_{to}$  is estimated based on the initial temperature distribution along the wellbore for all gridblocks. For other iterations, the value of  $U_{to}$  from the previous iteration is assigned to  $U_{to}$ . Once we have  $U_{to}$ , the inside-casing temperature and outside-tubing (or insulation) temperature can be calculated based on the fluid and wellbore temperatures. Because  $U_{to}$  is a function of these temperatures, its value can be recalculated and updated. If it is not close to its previous value, this loop will continue until appropriate convergence is achieved.
5. The superficial gas velocity is assigned to  $(v_{sg})_{k+1/2}$  from the previous iteration, the liquid superficial velocity is calculated from Equation (18) and the gas and liquid densities at  $k+1/2$  are approximated as  $(\rho_L)_{k+1/2} \approx (\rho_L)_k$  and  $(\rho_g)_{k+1/2} \approx (\rho_g)_k$ .
6. The gas in-situ volume fraction  $f_g$  is calculated based on multiphase flow equations (the type of equations is the user's selection) and other mixture properties, such as  $\rho_m$  are obtained. It should be noted that if the drift-flux model of Hasan et al.<sup>(23)</sup> is used as the multiphase flow equations in this step, it needs a friction factor that is a function of the unknown phase velocities. Therefore, after assuming some acceptable number for the friction factor, its computation is iterated until an appropriate convergence is achieved.
7. A new estimate of  $p_k^{sat}$ , using the momentum balance equation, is calculated. Right after that, temperature and all fluid properties are updated using EOS.
8. Heat loss is estimated using Equation (5).
9. New  $(v_{sg})_{k+1/2}$  is obtained from the energy balance equation using an iterative Newton-Raphson scheme. If it is different from the assumed one, Step 5 is repeated; otherwise, the convergence of pressure at the current gridblock is checked. If new  $p_k^{sat}$  that was obtained in Step 7 is different from the assumed one, Step 2 is repeated; otherwise, all previously described steps for the subsequent gridblocks are repeated.
10. Now,  $T_{wb|k}$  is updated by using  $\dot{Q}_{loss}$ , which was previously obtained in Step 8, and solves the formation Equation (7) or (8). Then the requested results are printed or saved in a text file. Next, one timestep is marched ahead. If the final time has been reached, the solution operation should be stopped; otherwise, all previous steps are repeated for the next timestep.

The solution algorithm with the assumptions previously explained is quite fast and stable. The reason is that the only formula where time appears explicitly is in Equations (7) or (8), which is solved by the fully implicit method. The model was run several times to investigate the effects of both wellbore gridblock and timestep sizes. The wellbore grid size was decreased from 400 ft to 5 ft (basecase) for each run. It was seen that a wellbore grid size of 100 ft caused a maximum relative error of 1.5% in steam quality calculation compared with the basecase. Reducing the wellbore grid size to less than 50 ft does not influence the results. Therefore, a wellbore grid size of 20 ft was chosen for all other runs. A series of other cases were run to see the effect of the timestep size. It was

**TABLE 1—FIELD DATA PARAMETERS FOR FIELD TEST 1A, 1B AND FIELD TEST 2**

	Field Test 1A and 1B	Field Test 2
$r_{ti}$	0.088500	0.083150
$r_{to}$	0.104167	0.098958
$r_{ins}$	No Insulation	No Insulation
$r_{ci}$	0.166667	0.166667
$r_{co}$	0.187500	0.187500
$r_{wb}$	0.600000	0.600000
$k_e$	1	1
$\alpha_e$	0.0286	0.0286
$g_T$	0.0283	0.0196
$k_{cem}$	0.2	0.2
$\varepsilon_{to}$	0.9	0.9
$\varepsilon_{ci}$	0.9	0.9
$\dot{W}$	4850	2800
$x$	0.8	0.8
$\rho_{wh}$	250	520
$T_{wh}$	50	71
Depth	1600	1700
Annulus pressure	14.7	14.7

found that the model can be run with the timestep as small as a fraction of a second and as large as a year without losing the solution stability. With a timestep size of an hour, a typical field model that will be explained later can take 65 seconds to run for a steam injection period of 100 hours.

## Results

Because of the sparseness of field data available in the literature that explains steam injection wells, only two sets of field data from Field Tests 1A and 1B [from a test on the 61–0 Martha Bigpond well<sup>(25)</sup>] and Field Test 2 [from a test on the 14–W Sallie Lee well<sup>(9)</sup>] are selected for our model validation. The field data are reported in **Table 1**. For Field Test 1A, the steam temperature is measured after 71 hours of the steam injection, and for Field Tests 1B and 2, the steam pressure is measured after 117 and 308 hours of the steam injection, respectively.

**Figs. 4–6** show the prediction of our model, the prediction of Fontanilla and Aziz's model<sup>(9)</sup> and the measured field data. For Field Test 1B, we also compare our results with those of Ali's model<sup>(8)</sup>, as depicted in **Fig. 7**. All of these figures indicate that our model is

in a good agreement with the field data. In **Figs. 8 and 9**, for Field Test 1A there are 3% and 4% differences between the lowest and highest predictions of the models at the bottomhole condition for the heat loss and steam quality, respectively. These numbers become 8% and 10% for Field Test 2 (**Figs. 10 and 11**). Because there is no field steam quality measurement, it is impossible to determine which model works better in this regard. Note that the flow regimes depicted in **Figs. 4 and 6** are predicted by our model using the multiphase drift-flux flow model of Hasan et al.<sup>(23)</sup>.

## Conclusions

In this paper, a numerical nonisothermal two-phase wellbore flow model has been developed and tested against both field data and the prediction of other models. This model entails the spatial and temporal discretization of the wellbore and formation domains and the solution of the discrete wellbore equations for mass, momentum and energy balance. A drift-flux model has been used to capture the slip phenomenon between the phases, and the time- and depth-dependent overall heat transfer coefficient has been incorporated into the model to capture the heat loss to the surroundings. The wellbore system has sequentially been solved with a double-iterative procedure, and the formation system has been solved in a fully implicit manner. Implementation of the drift-flux model of Hasan et al.<sup>(23)</sup> into our model for multiphase flow gives a good agreement between the prediction of our model and field data, but Beggs and Brill's method<sup>(24)</sup> for multiphase flow over-predicts the steam pressure and temperature.

It should be stressed that the type of model (analytical or numerical) adopted for the modelling of earth temperature (surrounding the wellbore) can have a significant effect on the accuracy of steam injection wellbore simulators. This aspect of the analysis is the subject of a current research study in our group<sup>(26)</sup>.

## Nomenclature

$A_g$  = area occupied by gas, ft<sup>2</sup>

$A_{ti}$  = inside tubing area, ft<sup>2</sup>

$C_o$  = distribution coefficient in the drift-flux model (dimensionless)

$C_{pe}$  = formation heat capacity, Btu/(lbm °F)

$d_{ti}$  = internal tubing diameter, ft

$f_g$  = gas insitu volume fraction (dimensionless)

$f_L$  = liquid insitu volume fraction (dimensionless)

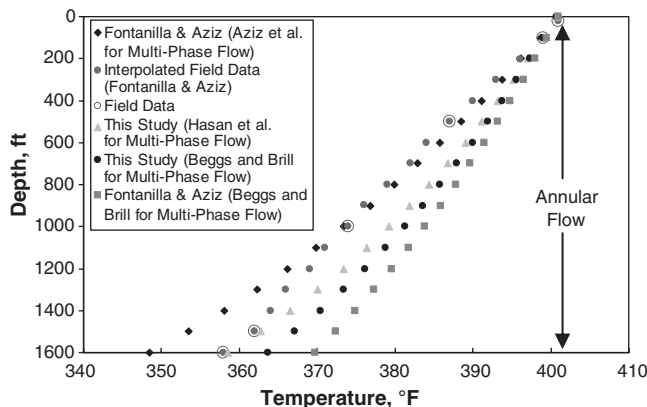
$f_m$  = Moody friction factor (dimensionless)

$g$  = acceleration caused by gravity, 32.17 ft/s<sup>2</sup>

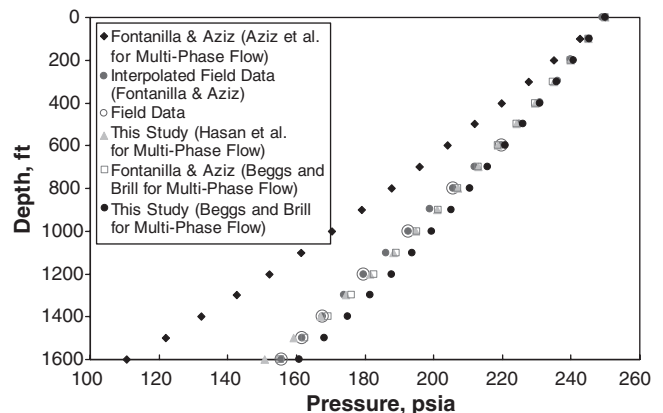
$g_c$  = gravitational conversion constant, 144 × g (lbm/lbf × in.<sup>2</sup>/ft<sup>2</sup> × ft/s<sup>2</sup>)

$g_T$  = geothermal gradient, °F/ft

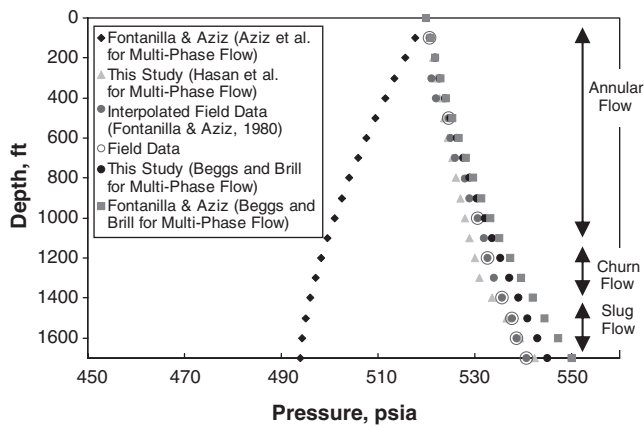
$h_c$  = convective heat transfer coefficient of fluid inside annulus, Btu/(hr ft<sup>2</sup> °F)



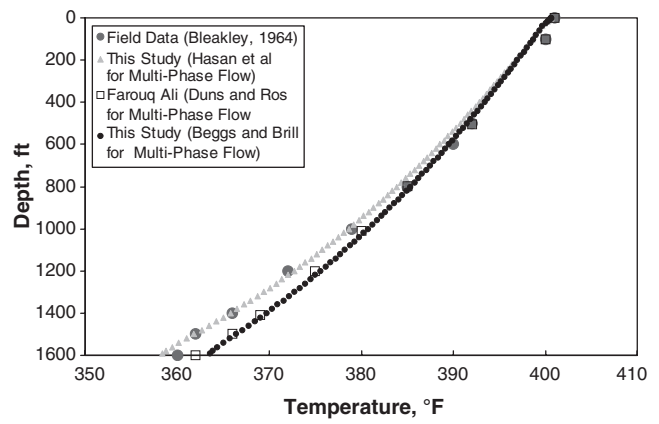
**Fig. 4: Validation of numerical results against Field Test 1A and comparison of predicted steam temperature with other models after 71 hours of steam injection.**



**Fig. 5: Validation of numerical results against Field Test 1B and comparison of predicted steam pressure with other models after 117 hours of steam injection.**



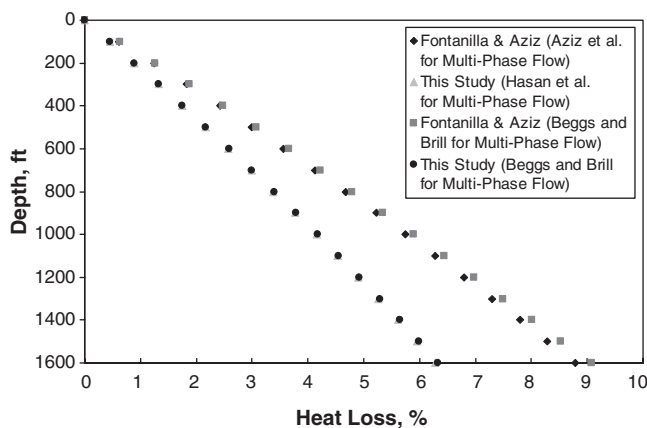
**Fig. 6: Validation of numerical results against Field Test 2 and comparison of predicted steam pressure with other models after 308 hours of steam injection.**



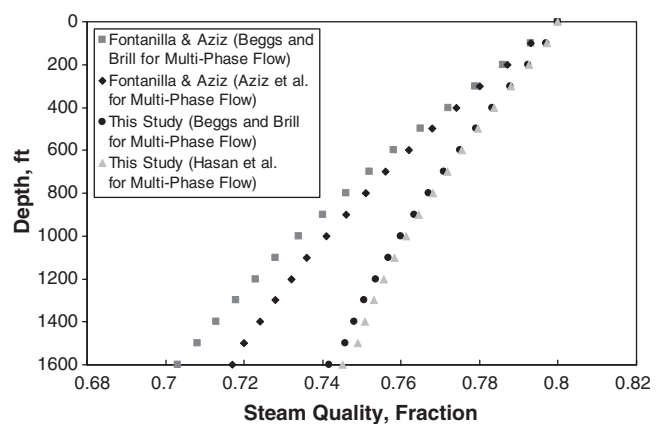
**Fig. 7: Validation of numerical results against Field Test 1B and comparison of predicted steam temperature with Farouq Ali's model(8) after 116 hours of steam injection [Farouq Ali in his paper (page 529) reported the injection time as 116 hours].**

$h_f$  = convective heat transfer coefficient of fluid, Btu/(hr ft<sup>2</sup> °F)  
 $h_g$  = gas enthalpy, Btu/lbm  
 $h_L$  = liquid enthalpy, Btu/lbm  
 $h_p$  = enthalpy of phase p (liquid water or steam), Btu/lbm  
 $h_r$  = radiative heat transfer coefficient of fluid inside annulus, Btu/(hr ft<sup>2</sup> °F)  
 $k_{cas}$  = thermal conductivity of casing, Btu/(hr ft °F)  
 $k_{cem}$  = thermal conductivity of cementing, Btu/(hr ft °F)  
 $k_e$  = thermal conductivity of formation, Btu/(hr ft °F)  
 $k_{er}$  = radial thermal conductivity of formation, Btu/(hr ft °F)  
 $k_{ez}$  = vertical thermal conductivity of formation, Btu/(hr ft °F)  
 $k_{ins}$  = thermal conductivity of insulation, Btu/(hr ft °F)  
 $k_t$  = thermal conductivity of tubing, Btu/(hr ft °F)  
 $L$  = total depth of well, ft  
 $p$  = wellbore pressure, psia  
 $\dot{Q}_{loss}$  = heat loss rate to surroundings, Btu/(hr ft)  
 $r$  = radius, ft  
 $r_{ti}$  = inside radius of tubing, ft  
 $r_{to}$  = outside radius of tubing, ft  
 $r_{ins}$  = radius of the outside insulation surface, ft  
 $r_{ei}$  = inside radius of casing, ft  
 $r_{co}$  = outside radius of casing, ft  
 $r_{wb}$  = cementing/formation interface radius, ft  
 $t$  = time, second (hour)  
 $T_f$  = fluid temperature inside tubing, °F

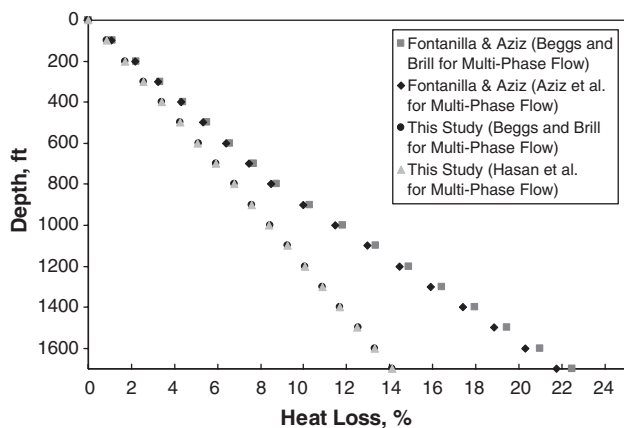
$T_e$  = formation temperature, °F  
 $T_{ei}$  = initial formation temperature, °F  
 $T_{eiwh}$  = initial wellhead temperature, °F  
 $T_{reservoir}$  = reservoir temperature, °F  
 $T_{surface}$  = surface temperature, °F  
 $T_{wb}$  = cementing/formation interface temperature (wellbore temperature), °F  
 $u_p$  = internal energy of phase p (liquid water or steam), Btu/lbm  
 $U_{to}$  = overall heat transfer coefficient, Btu/(hr ft<sup>2</sup> °F)  
 $v_g$  = gas velocity, ft/s  
 $v_L$  = liquid (water) velocity, ft/s  
 $v_m$  = mixture velocity, ft/s  
 $v_{sg}$  = superficial gas (steam) velocity, ft/s  
 $v_{sL}$  = superficial liquid (water) velocity, ft/s  
 $v_{\infty}$  = drift velocity of gas (steam) in liquid (water), ft/s  
 $\dot{w}$  = mass flow rate, lbm/hr  
 $x$  = steam quality, fraction  
 $z$  = wellbore direction, ft  
 $\alpha_e$  = formation thermal diffusivity, ft<sup>2</sup>/hr  
 $\Delta z$  = distance interval, ft  
 $\varepsilon_{ci}$  = emissivity of inside casing surface (dimensionless)  
 $\varepsilon_{to}$  = emissivity of outside tubing surface (dimensionless)  
 $\theta$  = local angle between well and the vertical direction, radian  
 $\rho_e$  = formation density, lbm/ft<sup>3</sup>  
 $\rho_g$  = gas density, lbm/ft<sup>3</sup>  
 $\rho_L$  = liquid density (water density), lbm/ft<sup>3</sup>  
 $\rho_m$  = mixture density, lbm/ft<sup>3</sup>



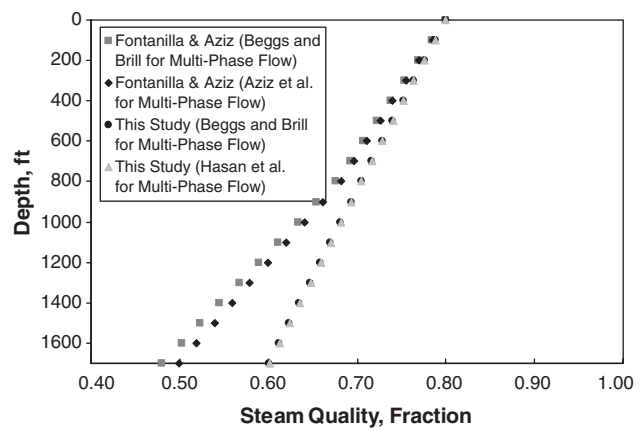
**Fig. 8: Comparison of predicted heat loss for Field Test 1A with other models after 71 hours of steam injection.**



**Fig. 9: Comparison of predicted steam quality for Field Test 1A with other models after 71 hours of steam injection.**



**Fig. 10: Comparison of predicted heat loss for Field Test 2 with other models after 308 hours of steam injection.**



**Fig. 11: Comparison of predicted steam quality for Field Test 2 with other models after 308 hours of steam injection.**

## Subscripts

$k$  = grid discretization index in the  $z$  direction  
 $p$  = phase  $p$ ,  $p=L$  (liquid) or  $p=g$  (gas)  
 $sat$  = saturation  
 $wh$  = wellhead

## Superscripts

$n$  = time discretization index

## SI Metric Conversion Factors

Btu $\times$ 1.055056	E+00 = kJ
$^{\circ}\text{F}$ $(^{\circ}\text{F}-32)/1.8$	= $^{\circ}\text{C}$
ft $\times$ 3.048*	E-01 = m
ft <sup>2</sup> $\times$ 9.290304*	E-02 = m <sup>2</sup>
lbf $\times$ 4.44822	E+00 = N
lbm $\times$ 4.535924	E-01 = kg
psia $\times$ 6.894757	E+00 = kPa


\*Conversion factor is exact

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